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Technical Notes

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Conductivity-Based Scheme for Identification of an Inner Pipe Boundary from Temperature Measurements

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Nomenclature

D	=	search direction of the conjugate gradient method
h	=	film coefficient
K	=	vector describing the circumferential distribution of the effective thermal conductivity
k_i	=	element of the vector **K**
k_{or}	=	thermal conductivity of the original pipe wall
m	=	number of measurement point
n	=	iteration number
n	=	normal vector
p	=	distance between the pipe center to the discrete point of inner pipe boundary
r	=	radius in the polar coordinate
r_1	=	inner radius of the domain Ω_2
r_2	=	outer radius of the domain Ω_2
T	=	temperature
T_a	=	ambient temperature
T_f	=	temperature of the fluid flowing through the pipe
β	=	search step size of the conjugate gradient method
ε	=	small positive number
ζ	=	pipe wall boundary
θ	=	angle in the polar coordinate
σ	=	standard deviation of the temperature measurement
Ω	=	computational domain

Subscripts

in	=	inner boundary of the pipe wall
out	=	outer boundary of the pipe wall

Superscript

n	=	iteration number

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I. Introduction

CORROSION, as is known to us all, is the key reason for the wall thinning of pipe; therefore, the timely quantitative identification of a pipe's inner-surface condition is of great importance [1]. In most developed methods (such as the steepest-descent method [2], the Levenberg–Marquardt method [3], and the conjugate gradient method [3,4]), the boundary shape must be updated during the iteration process until the convergence of the calculated and the measured temperature distributions of the inspection surface. That is to say, the remeshing of grid for the new irregular-shaped calculation domain is needed for each iteration, which may greatly increase the complexity of the algorithm and also the time used for the solution of the inverse problem.

In this Note, a new computational scheme based on the conjugate gradient method is presented for the identification of the inner boundary shape by estimating the circumferential distribution of the effective thermal conductivity of the pipe wall. Because the iterative variable is conductivity, remeshing of the grid is no longer needed; moreover, the identification work can also be conducted simply in a regular-shaped domain. The feasibility and effectiveness of this scheme will be presented in a series of simplified two-dimensional numerical examples.

II. Mathematical Formulation of the Problem

The cross-sectional schematic of the pipe is shown in Fig. 1a. Hot fluid circulates through the pipe when the temperature distribution is recorded at the outer pipe surface (i.e., the inspection surface), from which heat is dissipated into the surroundings by natural convection. For simplification, a constant film coefficient h_{out} is adopted for the inspection surface in this Note. The inverse heat conduction problem can be mathematically described by the following governing steady-state equation

$$\frac{\partial}{\partial r}\left(k_{\text{or}} r \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{k_{\text{or}}}{r}\frac{\partial T}{\partial \theta}\right) = 0 \tag{1}$$

and boundary conditions:

$$\begin{cases} -k_{\text{or}}\left(\frac{\partial T}{\partial \mathbf{n}_{\text{in}}}\right)_{\varsigma_{\text{in}}} = h_{\text{in}}(T_{\text{in}} - T_f) \\ -k_{\text{or}}\left(\frac{\partial T}{\partial \mathbf{n}_{\text{out}}}\right)_{\varsigma_{\text{out}}} = h_{\text{out}}(T_{\text{out}} - T_a) \end{cases} \tag{2}$$

The goal of the identification problem is to identify the inner pipe boundary shape $p(\theta)$ based on the temperature measurements of the inspection surface.

III. Description of the Computational Scheme

The inner pipe boundary shape in this computational scheme will not be identified directly, but based on the estimation of the circumferential distribution of the effective thermal conductivity of the pipe wall, which is shown schematically in Fig. 1b. When estimating the conductivity, the inner pipe boundary is deemed as the original circular boundary before corrosion took place, and the method employed is the conjugate gradient method, along with the finite volume method. More details are presented in the following subsections.

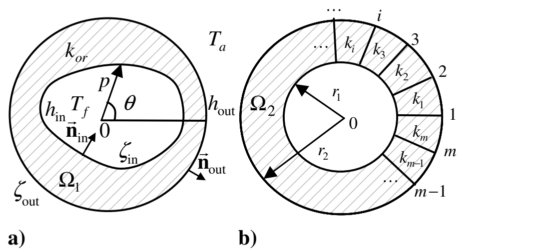

Fig. 1 Schematic of the transfer of computational domain ($\Omega_1 \rightarrow \Omega_2$).

A. Conjugate Gradient Method for the Estimation of Thermal Conductivity Distribution

The iteration function of the conjugate gradient method for estimating the circumferential distribution of the effective thermal conductivity shown in Fig. 1b can be expressed as [3,4]

$$\mathbf{K}^{n+1}(\theta) = \mathbf{K}^n(\theta) - \beta^n \mathbf{D}^n \tag{3}$$

On the calculation of the terms β^n and $\mathbf{D}^n$, one can refer to [4].

B. Stopping Criterion

The stopping criterion for iteration function (3) is

$$J[\mathbf{K}(\theta)] = \sum_{i=1}^{m}\left[T_i^n - T_{\text{out}i}\right]^2 < \varepsilon \tag{4a}$$

where T_i represents the calculated temperature distribution of the inspection surface, and T_{oi} denotes the original measured temperature distribution. When the random temperature measurement error is considered, the discrepancy principle [4] is used for calculating the stopping criterion:

$$\varepsilon = m\sigma^2 \tag{4b}$$

C. Calculation of the Inner Boundary Shape Based on the Conductivity Estimation Result

The inner pipe boundary shape $p(\theta)$ can be calculated based on the final estimation of the circumferential distribution of the effective thermal conductivity of the pipe wall according to the following equation:

$$p_i = r_2 e^{-[\frac{k_{\text{or}}}{k_i}\ln(\frac{r_2}{r_1})]} \tag{5}$$

where k_i is the ith element of the estimated conductivity distribution vector $\mathbf{K}(\theta)$ in Fig. 1b. The preceding function can be deduced easily based on the equality between the radial thermal resistances of one element shown in Fig. 1b and the corresponding part in Fig. 1a.

D. Computational Procedure

1) Give an initial guess of the inner pipe boundary shape, and calculate the initial guess of the conductivity distribution $\mathbf{K}^1(\theta)$ according to Eq. (5).

2) Solve the temperature distribution of the outer pipe surface in domain Ω_2 based on the finite volume method.

3) Check the stopping criterion given by Eq. (4a) or Eq. (4b), and go to step 5 if satisfied.

4) Compute the new conductivity distribution $\mathbf{K}^{n+1}(\theta)$ based on Eq. (3), and return to step 2.

5) Compute the inner pipe boundary shape based on the identification result of the conductivity distribution according to Eq. (5).

IV. Numerical Examples and Discussion

In all test cases, we have chosen $T_f = 150°\text{C}$, $T_a = 25°\text{C}$, $h_{\text{in}} = 1000\ \text{W}/(\text{m}^2\text{K})$, $h_{\text{out}} = 10\ \text{W}/(\text{m}^2\text{K})$, $r_1 = 0.08$ m, $r_2 = 0.1$ m, and $m = 48$. The inner pipe boundary shapes of the two test cases to be determined are as follows:

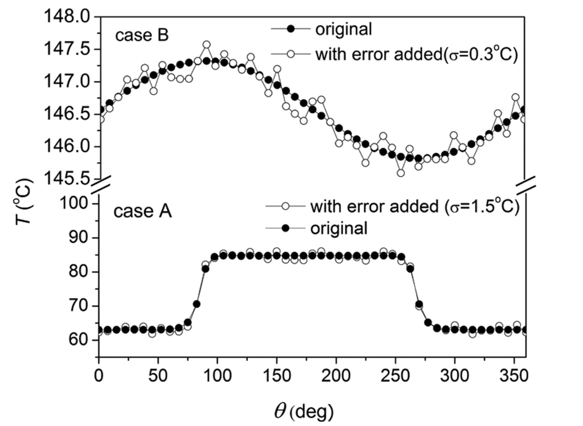

Fig. 2 Simulated temperature measurements with and without random measurement error added for the two cases.

Case A:

$$p(\theta) = \begin{cases} 0.09\ \text{m} & 0.5\pi < \theta < 1.5\pi \\ 0.08\ \text{m} & 0 \le \theta \le 0.5\pi;\ 1.5\pi \le \theta \le 2\pi \end{cases} \tag{6}$$

Case B:

$$p(\theta) = 0.085 + 0.005\sin(\theta)m \qquad 0 \le \theta \le 2\pi \tag{7}$$

The conductivities of the pipe wall are 0.1 and 10.0 W/(mK) for case A and case B, respectively.

The measured temperature distributions on the inspection surface are simulated by the solutions of the multidimensional Eq. (1) based on the inner boundary shapes in Eqs. (6) and (7), to which errors can also be added to study the effect of measurement error on the identification result. In this Note, Eq. (1) is solved by the finite element method with 2160 linear triangular elements in domain Ω_1, and the results are plotted in Fig. 2. From the figure, one can initially estimate the inner boundary shape. However, the inverse method must be developed for the quantitative identification, because the exact size and shape of the irregular-shaped inner boundary are difficult to determine, due to the heat diffusion within the pipe wall.

Figure 3 shows the identification results of the inner pipe boundary shape for the two test cases when $\varepsilon = 1.0$. Good agreement between the estimated and the exact inner pipe boundary shapes can be obtained. The initial guess has a negligible effect on the identification results.

As shown in Fig. 4, when the random measurement error is considered, reliable identification results can still be obtained. For case A, the measurement error when $\sigma = 1.5°\text{C}$ is about 2.0% of the average temperature of the outer pipe surface, and the largest average relative error of the identification results is 1.0%; therefore, the

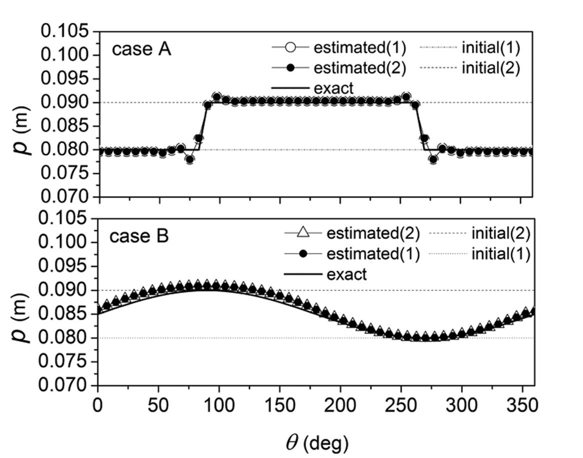

Fig. 3 Identification results for different initial guesses when no measurement error is considered for the two cases ($\varepsilon = 1.0$).

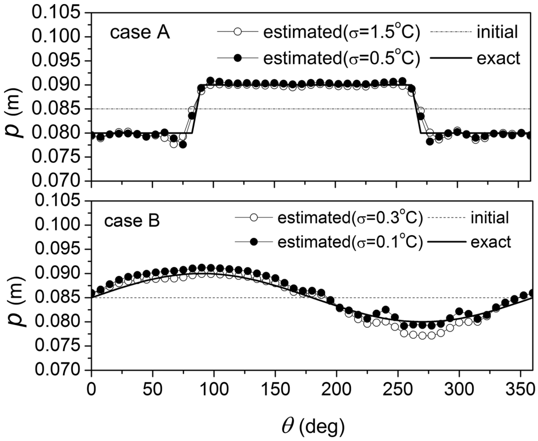

Fig. 4 Identification results when random temperature measurement error is considered for the two cases.

computational scheme does not magnify the error induced by the measurement. For test case B, It should be mentioned that only the effect of the measurement error with $\sigma \leq 0.3°C$ is discussed here, because the measurement error (which is nearly as large as the maximum temperature difference of the inspection surface) will make the inner pipe boundary shape fall out of the range that can be detected from the outer pipe surface, and hence the quantitative identification work is no longer needed. For case B, one also can see that the measurement error considered has a negligible effect on the identification results, because the largest average relative error is still less than 1.7% [3]. The discrepancies at the edges for test case A are caused by the heat diffusion in the pipe wall and also by the temperature measurement error when the error is added. In practical work, an averaged result of multiple estimations based on multiple measurements will be more precise. It should also be mentioned that the scheme is very fast, because all the identification results in this section are obtained within 60 s.

V. Conclusions

By solving two-dimensional inner pipe boundary identification problems, the numerical experiments have certified the feasibility and effectiveness of the presented computational scheme. The effect of the factors such as the initial guess and the temperature measurement error is negligible. Because no updating of the boundary shape is needed during the iteration process, the method is simple and timesaving; therefore, this computational scheme is very promising in the practical inspection and identification work. Of course, in the application, there are still many factors, such as the uncertainties in the boundary conditions and in the fluid temperature, that will also affect the identification results and that need to be discussed. This is our future work.

Acknowledgment

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